

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



Precision predictions for Higgs production

Claude Duhr

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05/06/2013

Introduction

- ATLAS and CMS have discovered a Higgs boson.
 - ➔ So far the signal strength is compatible with the Standard Model (SM).
 - ➔ New physics could appear via quantum corrections.

Introduction

- ATLAS and CMS have discovered a Higgs boson.
 - ➔ So far the signal strength is compatible with the Standard Model (SM).
 - ➔ New physics could appear via quantum corrections.
- Current **experimental** precision on cross section: $\sim 30\%$.
- Status of **theoretical** predictions:

	σ [8 TeV]	$\delta\sigma$ [%]
LO	9.6 pb	$\sim 25\%$
NLO	16.7 pb	$\sim 20\%$
NNLO	19.6 pb	$\sim 9\%$
N3LO	???	$\sim 4\%$

➔ At some point, experimental precision will go below the theoretical uncertainty.

Introduction

- Goal of this project:

Compute the partonic Higgs cross section to next-to-next-to-next-leading order (N³LO) in perturbative QCD!

- Challenge: Never before has an N³LO computation for a hardon collider been attempted ...

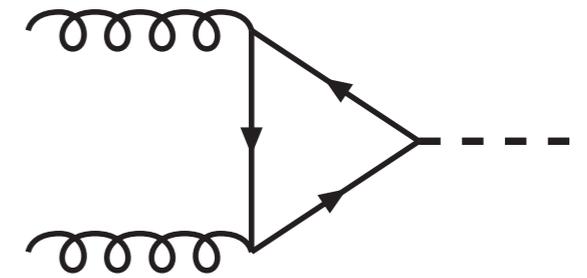
- ➔ Uncharted territory!

- ➔ New conceptual challenges.

The gluon fusion cross section

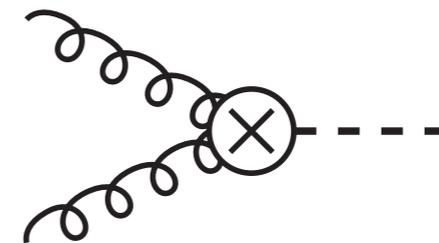
- The dominant Higgs production mechanism at the LHC is gluon fusion.

→ Loop-induced process.



- For a light Higgs boson, the top quark can be integrated out.
- As a result, we obtain a dimension five operator describing a tree-level coupling of the gluons to the Higgs boson:

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 H G_{\mu\nu}^a G_a^{\mu\nu}$$

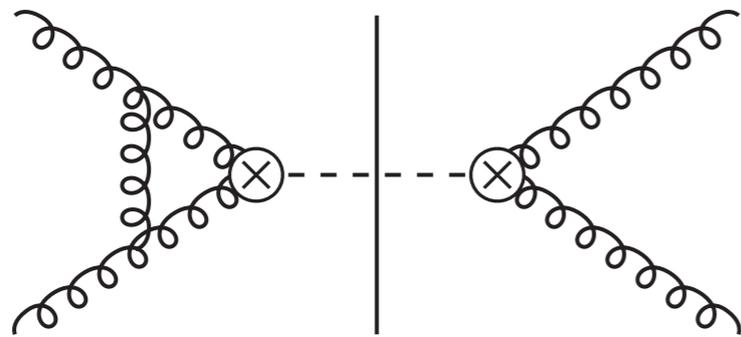


- In the rest of the talk, I will only concentrate on the effective theory.

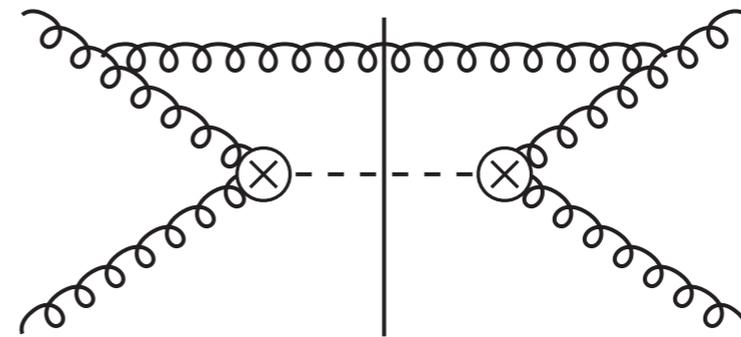
The gluon fusion cross section

- At NLO, there are two contributions (~1991):

[Dawson; Djouadi, Spira, Zerwas]



Virtual corrections ('loops')

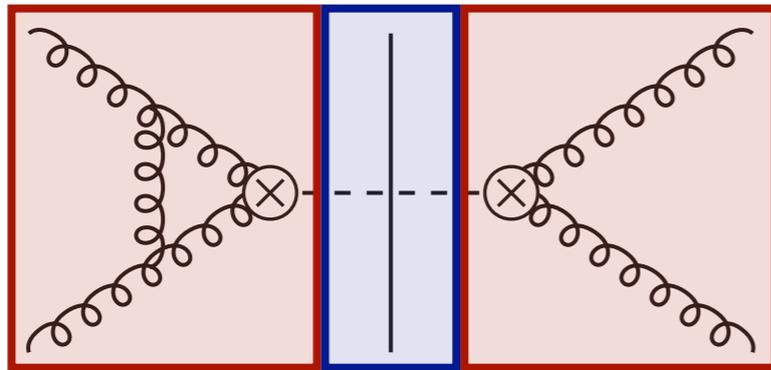


Real emission

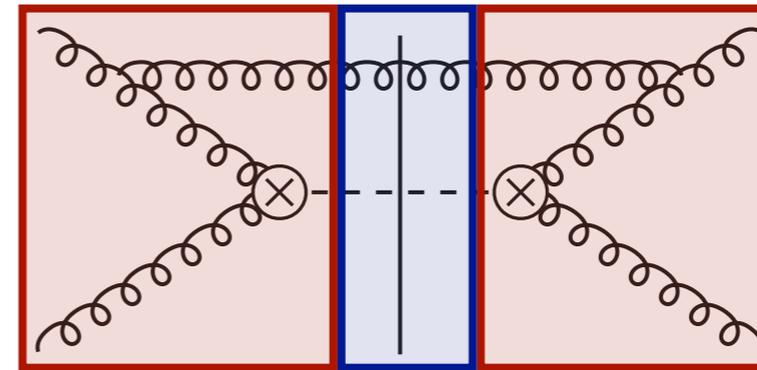
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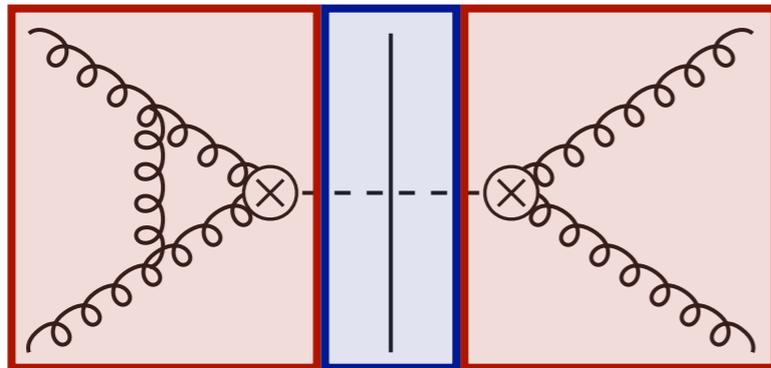
Real emission

$$\int d\Phi \mathcal{M}_i \mathcal{M}_j^*$$

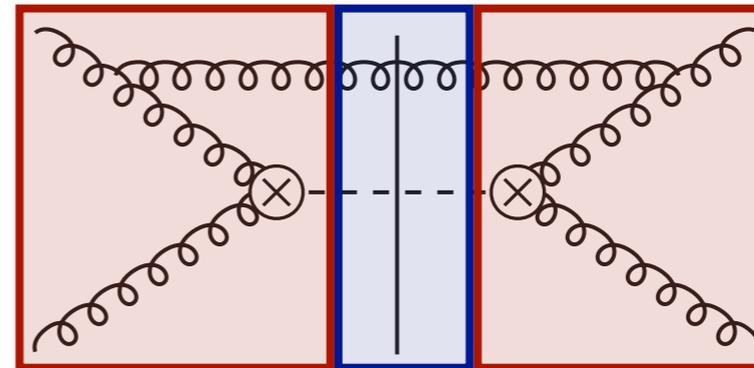
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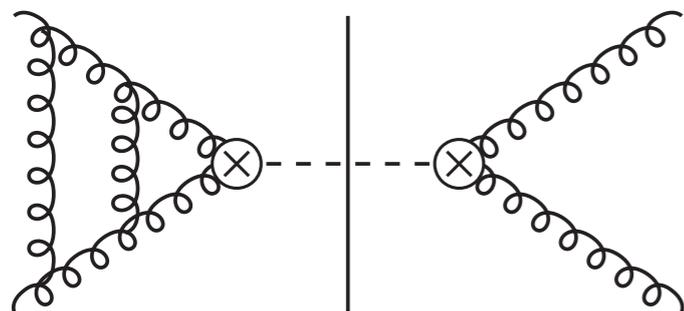
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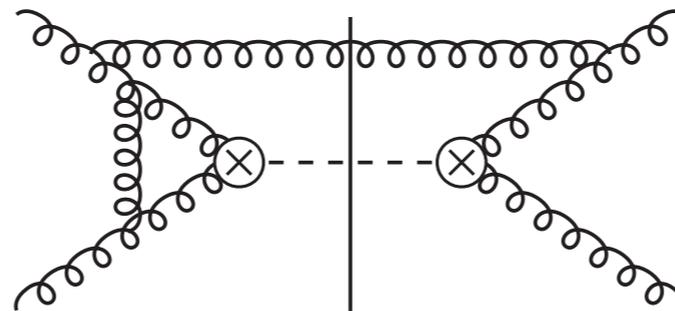
Real emission

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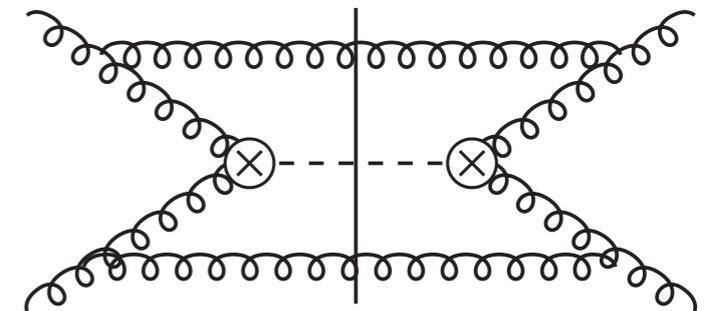
[Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]



Double virtual



Real-virtual

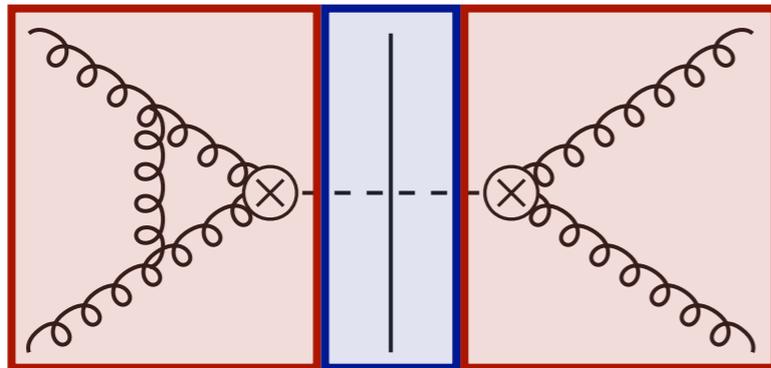


Double real

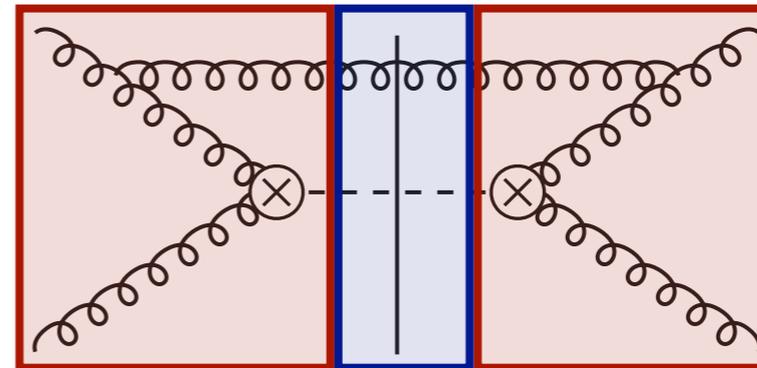
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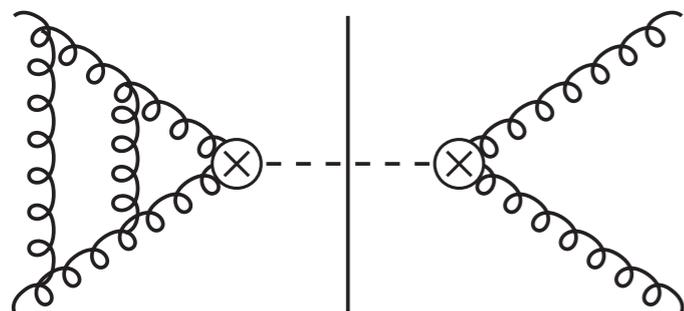
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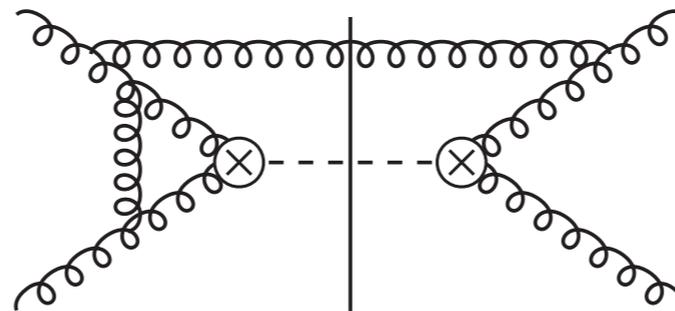
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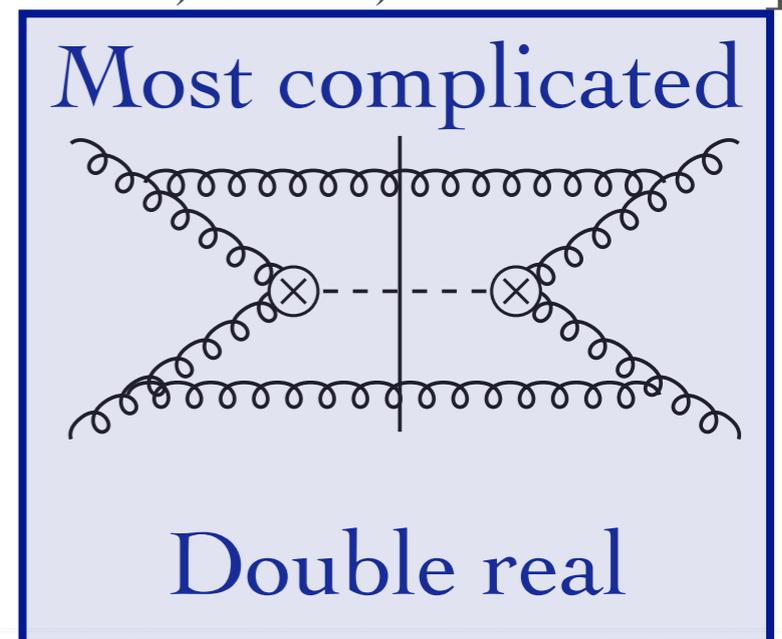
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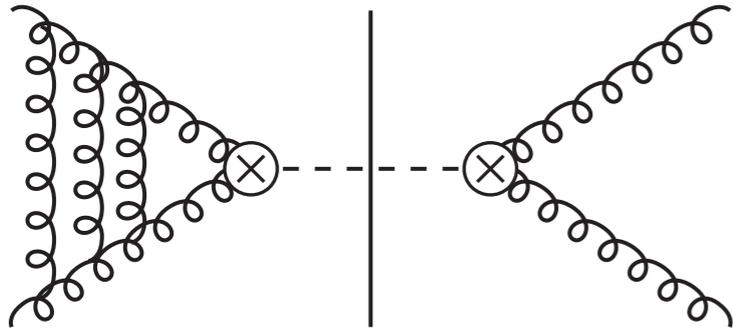
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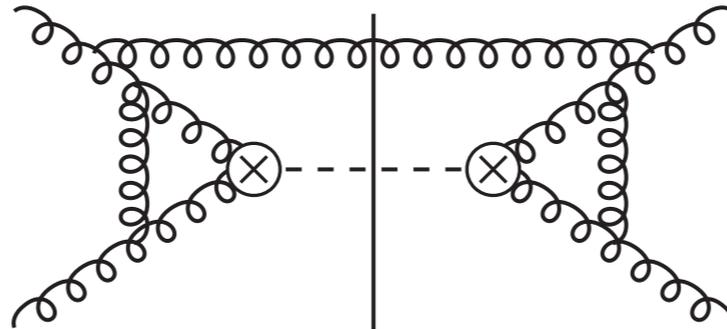
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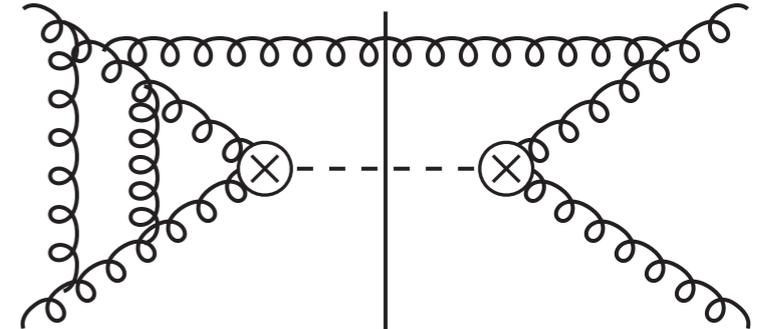
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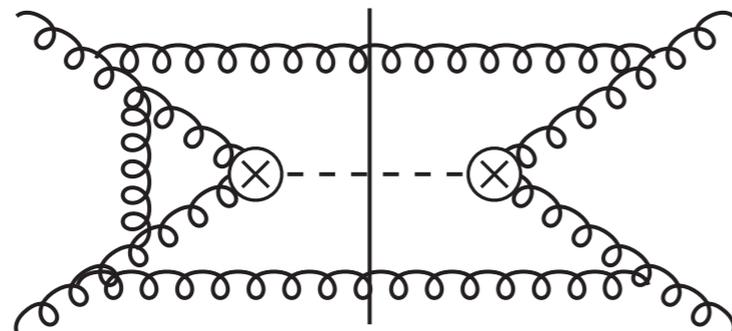
Triple virtual



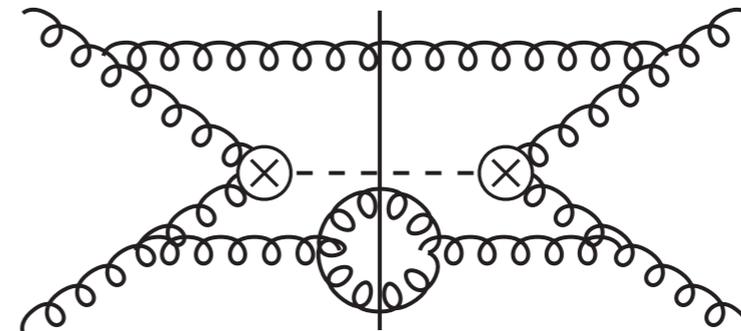
Real-virtual squared



Double virtual real

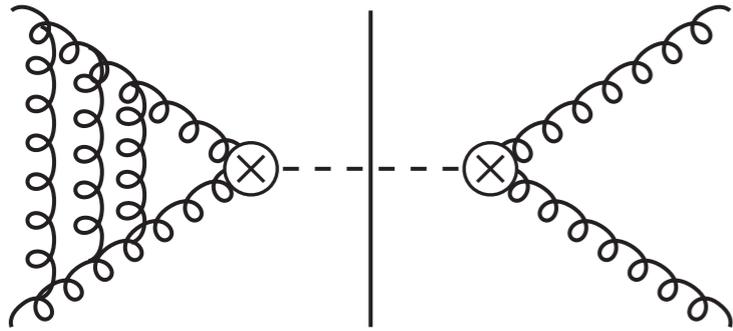


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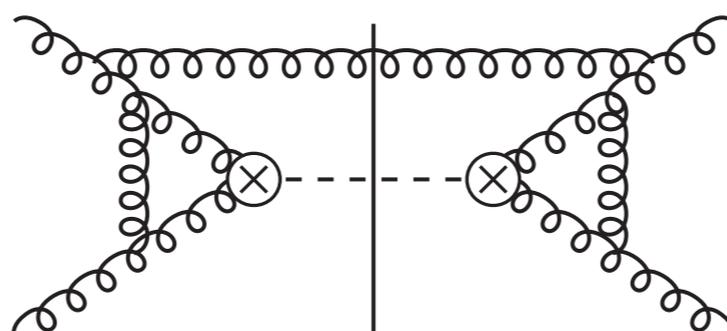


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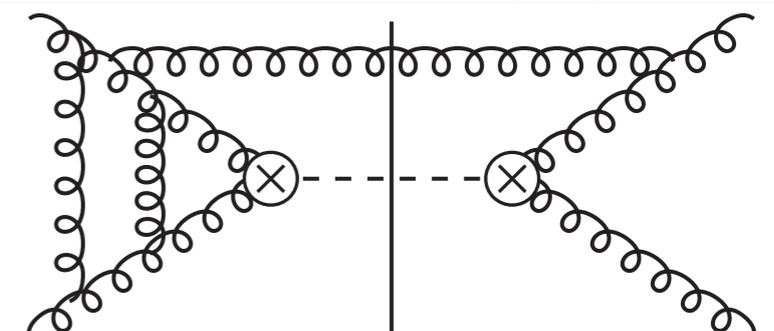
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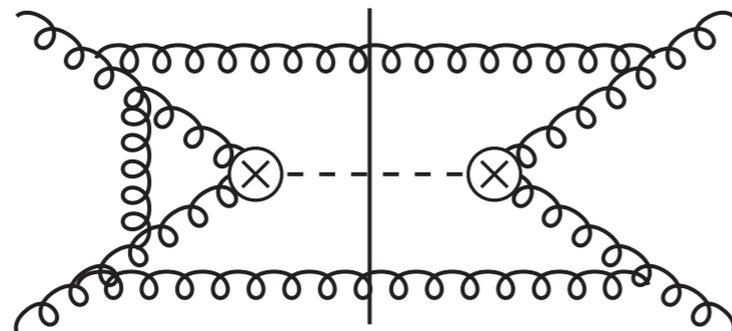


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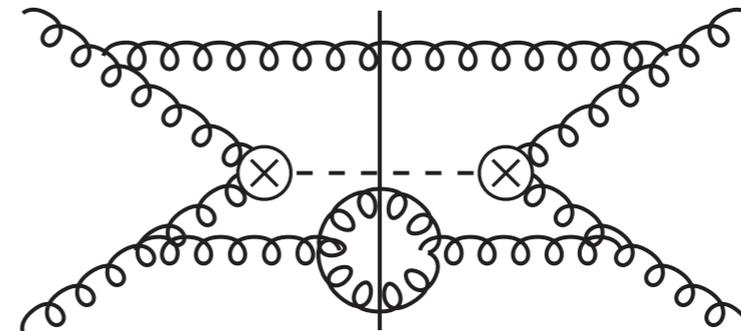


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[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]



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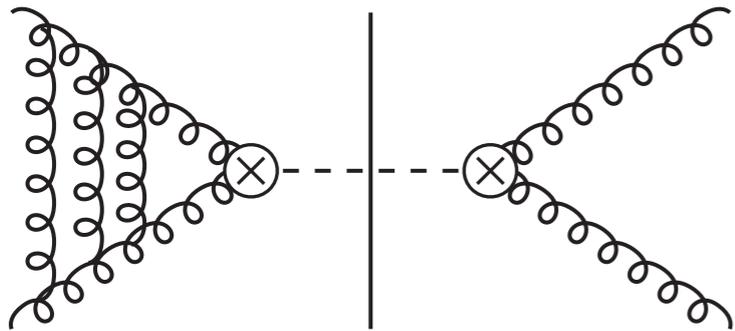


✗ Triple real

✓ + Convolution with splitting functions.

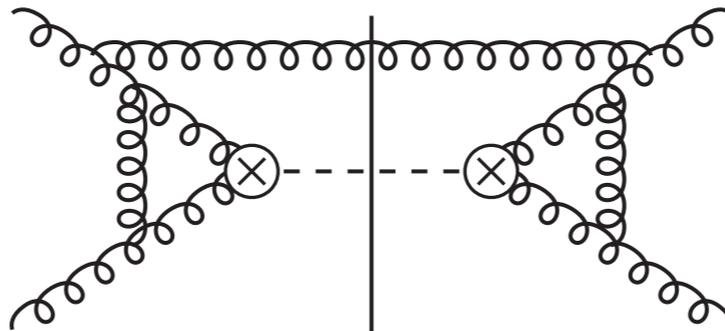
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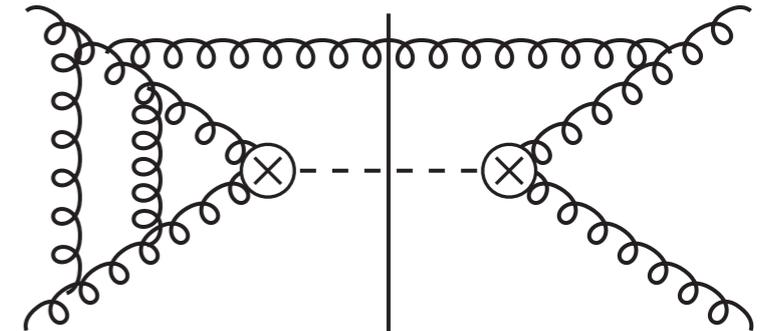


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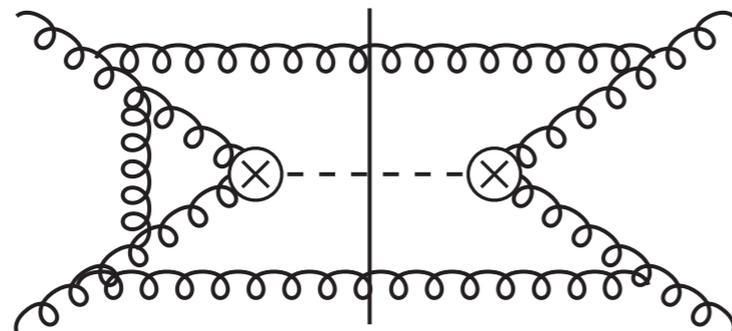
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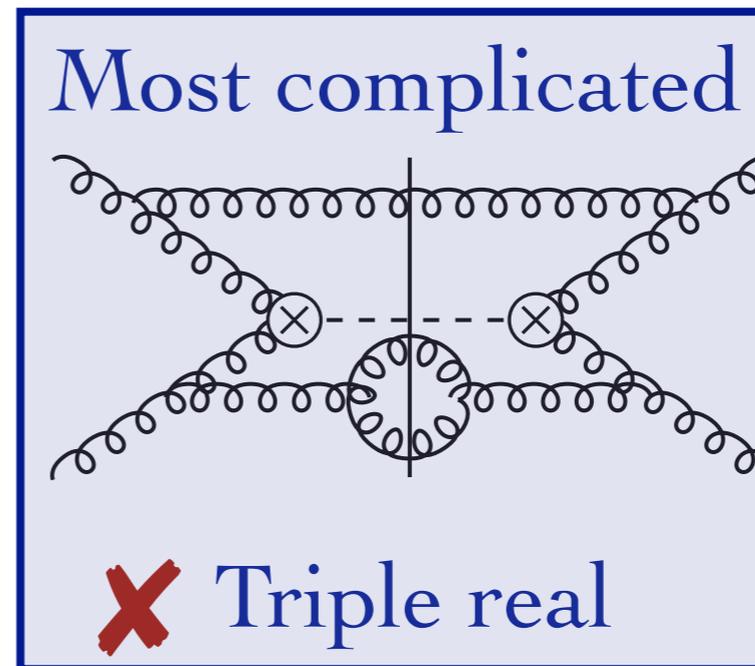
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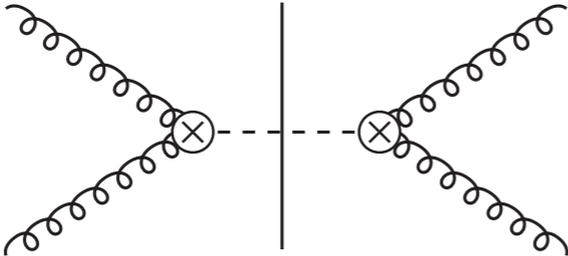
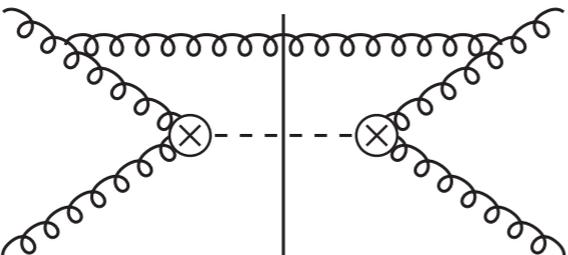
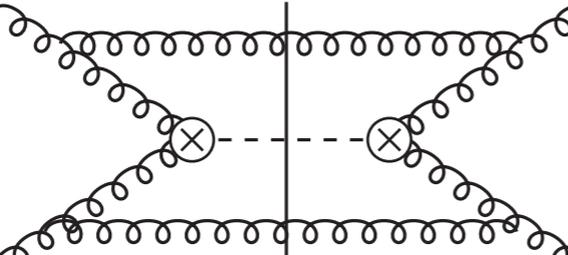
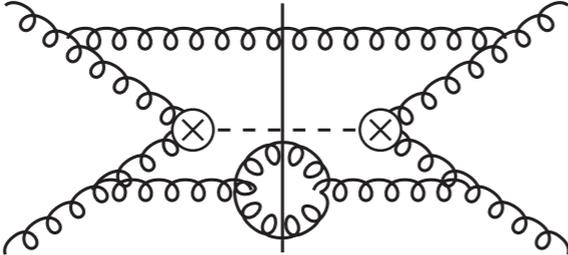


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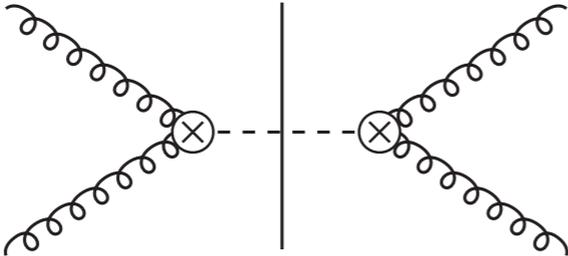
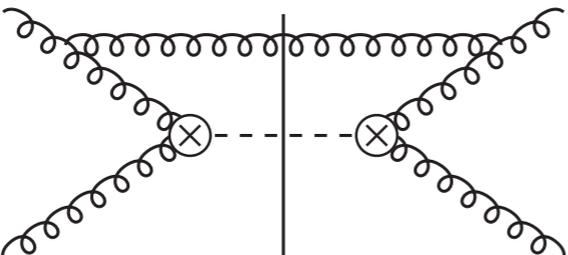
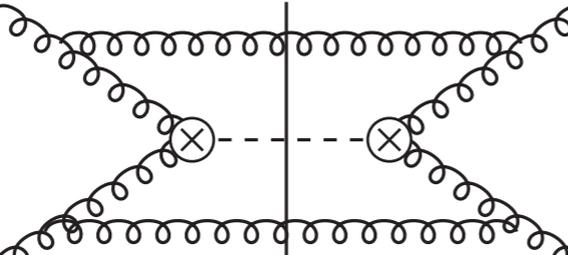
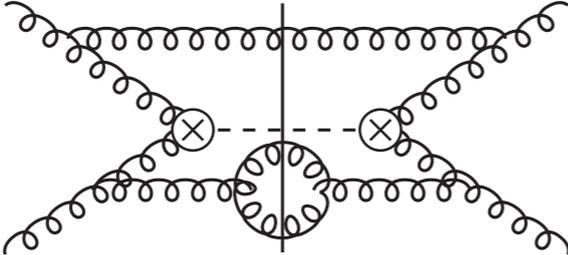
Reverse-unitarity @ N3LO

Growth in complexity for real emission

LO		1 diagram	1 integral
NLO			
NNLO			
N3LO			

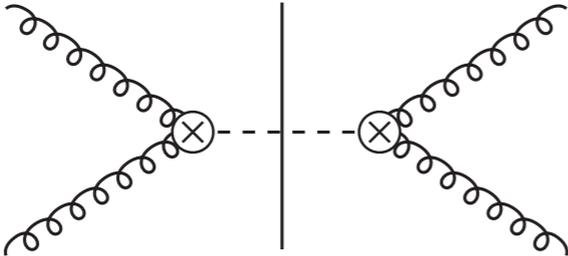
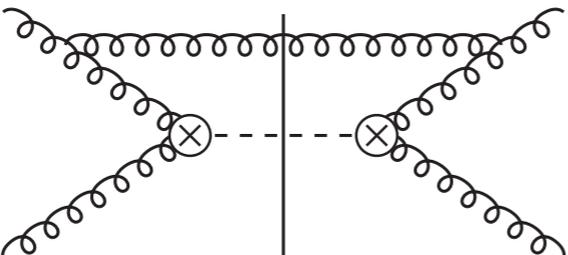
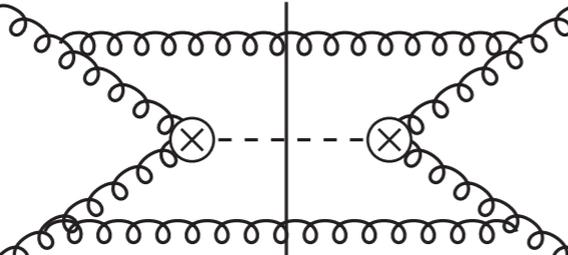
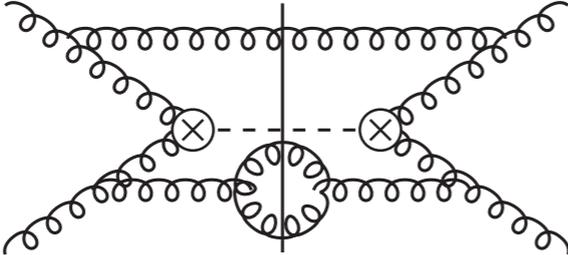
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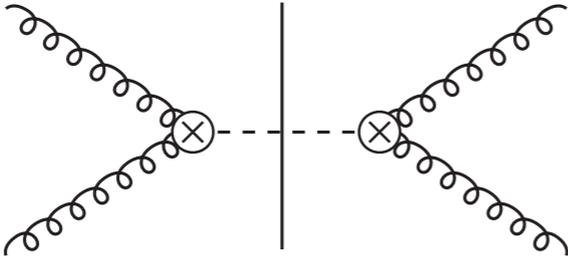
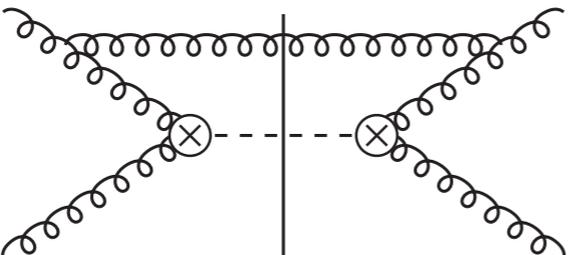
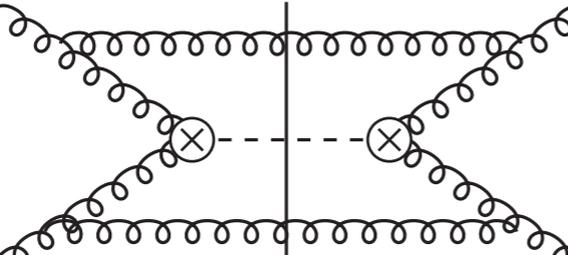
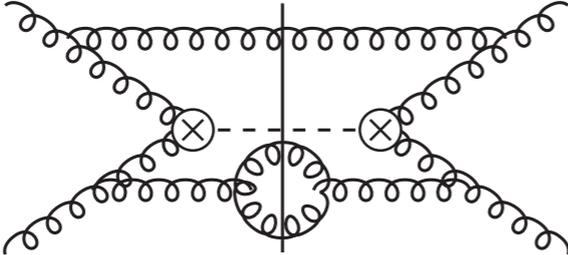
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N3LO			

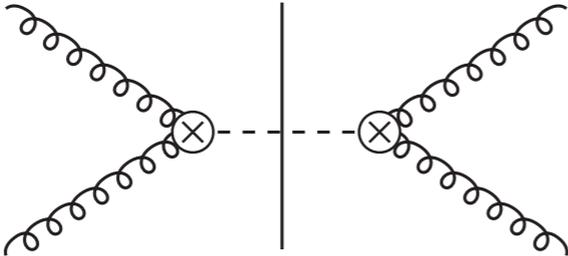
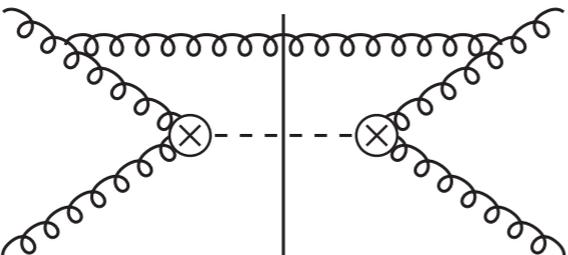
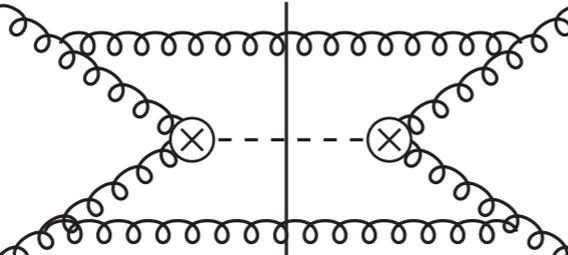
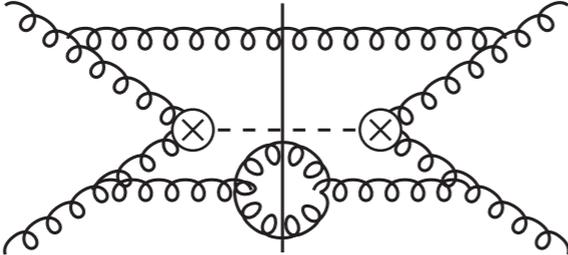
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Need new technology!

Reverse-Unitarity

- Optical theorem:

$$\text{Im} \text{ (loop diagram) } = \int d\Phi \text{ (cut diagram)}$$

- ➔ Discontinuities of loop amplitudes are phase space integrals.

Reverse-Unitarity

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$$\text{Im} \text{ (loop diagram) } = \int d\Phi \text{ (cut diagrams)}$$

- ➔ Discontinuities of loop amplitudes are phase space integrals.
- **Reverse-Unitarity:** write inclusive phase space integrals as unitarity cuts of loop integrals.
 - [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]
 - ➔ Makes inclusive phase space integrals accessible to all the technology developed for multi-loop computations!
 - ➔ Reverse-unitarity applied successfully at NNLO.
 - [Anastasiou, Melnikov]
 - ➔ Need something new to deal with 200 integrals at N3LO...

Reverse-unitarity @ N3LO

- **Idea:** Perform the expansion around threshold expansion!

➔ Known to give very good results at NNLO. [Harlander, Kilgore]

$$\sigma(z) = \bar{z}^{-1-6\epsilon} \sigma_{-1} + \bar{z}^{-6\epsilon} \sigma_0 + \mathcal{O}(\bar{z}) \quad \bar{z} = 1 - \frac{m^2}{s}$$

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➔ Which diagrams contribute to which order?

➔ Integrals can contribute to many orders.

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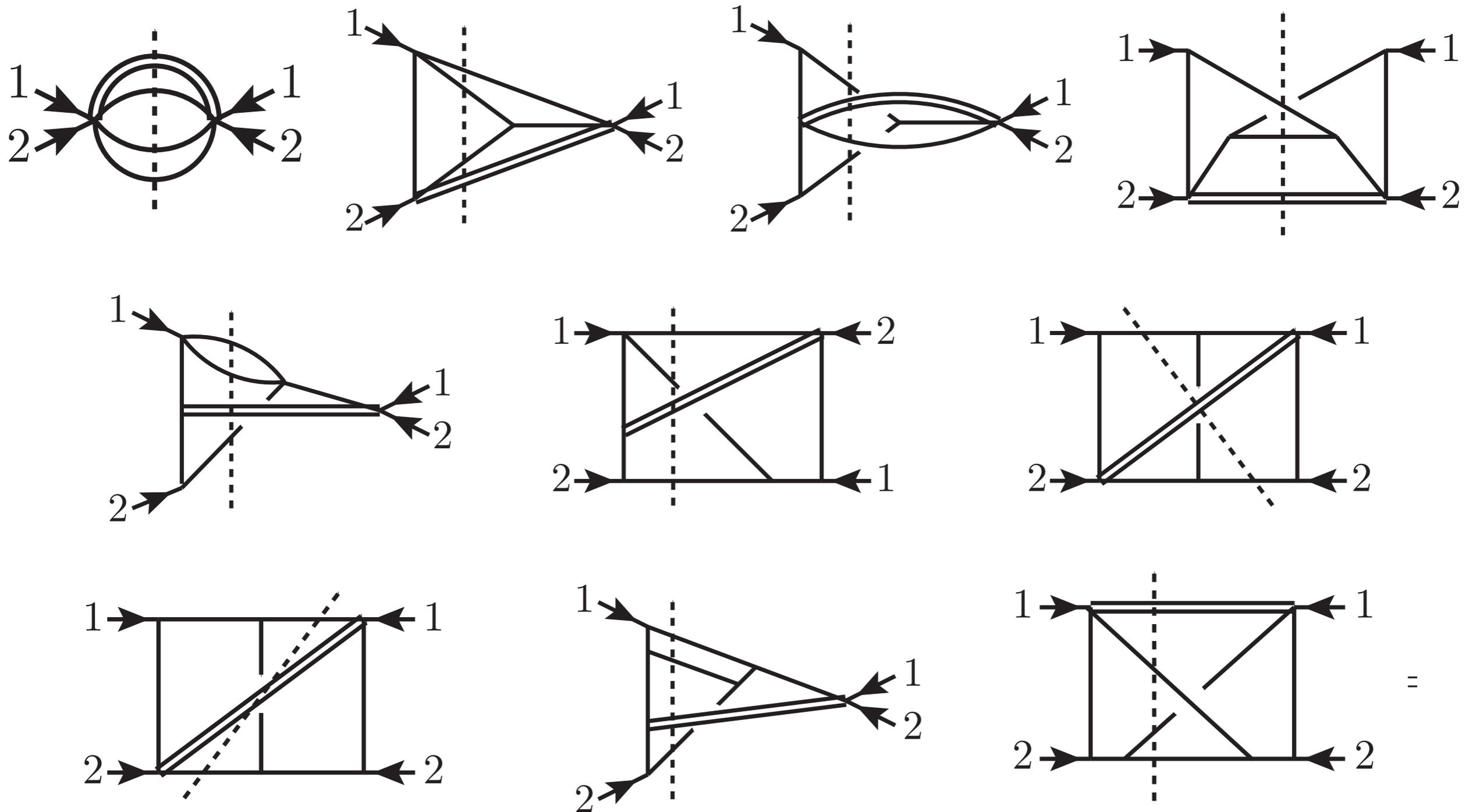
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- **Problems:**

- ➔ Which diagrams contribute to which order?
 - ➔ Integrals can contribute to many orders.
- We have developed a way to combine reverse unitarity with the threshold expansion for real-emission diagrams.
 - ➔ Number of integrals reduced from 200 to 10 for the first two terms in the threshold expansion.
 - ➔ Some integrals needed input from modern number theory to be computed

Soft triple real emissions



[Anastasiou, Dulat, CD, Mistlberger]

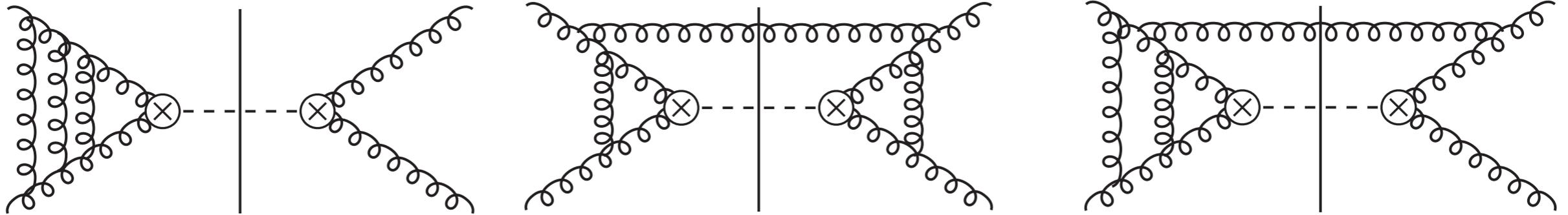
Soft triple real cross sections

- The integrals immediately allow us to write down the first two terms in the soft expansion of the cross section, e.g.,

$$\begin{aligned}
 \sigma_{gg \rightarrow H+gq\bar{q}}^{S(0)} &= \frac{2^5}{3^7} \frac{1}{8(N_c^2 - 1)^2} (4\pi\alpha_S)^3 \Phi_4^S(\epsilon) C_A C_F c_H^2 N_f \\
 &\times \left\{ \frac{153090}{\epsilon^4} - \frac{1604043}{\epsilon^3} + \frac{1}{\epsilon^2} (-29160\zeta_2 + 4903902) \right. \\
 &+ \frac{1}{\epsilon} (-204120\zeta_3 + 321732\zeta_2 - 4833675) - 874800\zeta_4 + 2252124\zeta_3 - 911088\zeta_2 \\
 &+ 203535 + \epsilon(-2711880\zeta_5 - 233280\zeta_2\zeta_3 + 9651960\zeta_4 - 6290136\zeta_3 - 492210\zeta_2 \\
 &+ 1667109) + \epsilon^2(-9360360\zeta_6 - 816480\zeta_3^2 + 29921076\zeta_5 + 2573856\zeta_2\zeta_3 \\
 &- 26589060\zeta_4 - 4323186\zeta_3 + 4693212\zeta_2 + 1294731) \\
 &+ 2C_A C_F \left[\frac{167670}{\epsilon^4} - \frac{1743039}{\epsilon^3} + \frac{1}{\epsilon^2} (-29160\zeta_2 + 5267592) + \frac{1}{\epsilon} (-204120\zeta_3 \right. \\
 &+ 321732\zeta_2 - 5183163) - 874800\zeta_4 + 2252124\zeta_3 - 911088\zeta_2 + 337959 \\
 &+ \epsilon(-2711880\zeta_5 - 233280\zeta_2\zeta_3 + 9651960\zeta_4 - 6290136\zeta_3 - 492210\zeta_2 + 1651749) \\
 &+ \epsilon^2(-9360360\zeta_6 - 816480\zeta_3^2 + 29921076\zeta_5 + 2573856\zeta_2\zeta_3 - 26589060\zeta_4 \\
 &\left. - 4323186\zeta_3 + 4693212\zeta_2 + 1284491) \right] + \mathcal{O}(\epsilon^3) \left. \right\}.
 \end{aligned}$$

[Anastasiou, Dulat,
CD, Mistlberger]

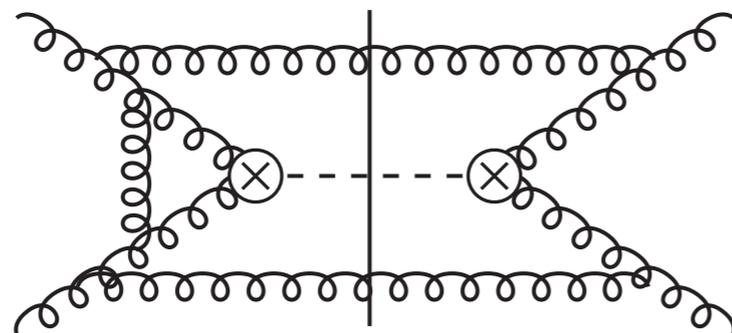
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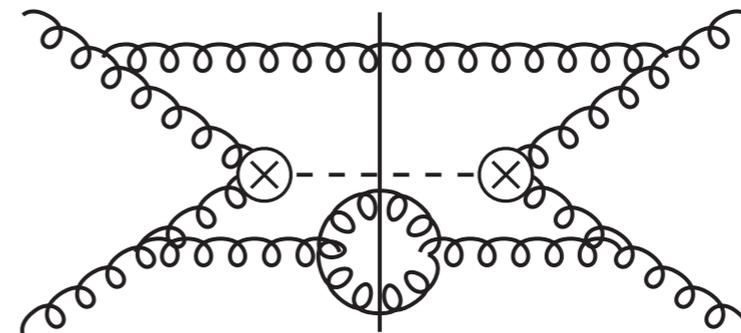
✓ Triple virtual

✗ Real-virtual squared

✗ Double virtual real



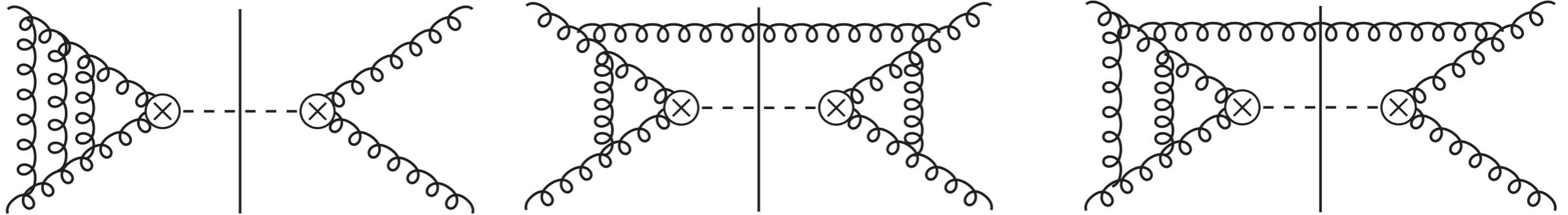
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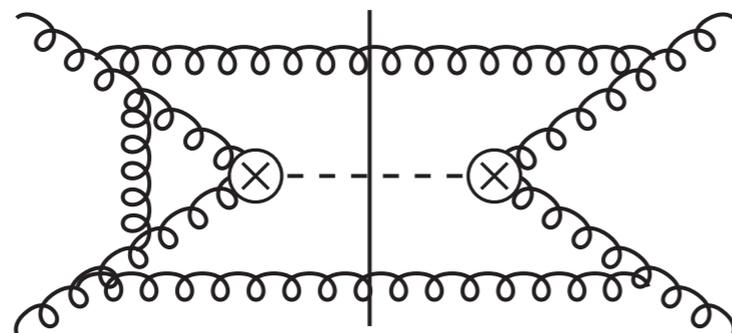
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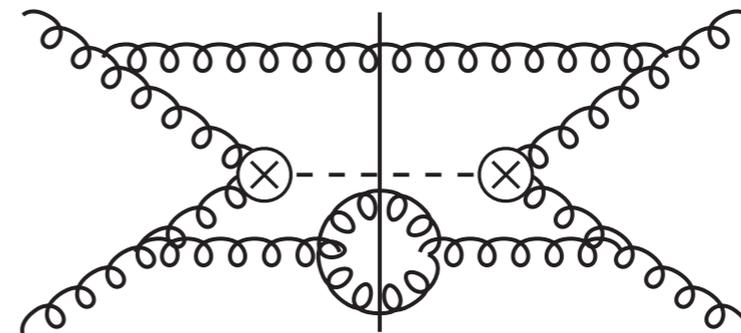
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Conclusion

- We have developed a new technique to compute the threshold expansion of real emission diagrams.
- Application to Higgs@N³LO has allowed us to obtain the first two terms in the soft expansion of the triple real contribution.
- Extension of this approach to mixed real-virtual integrals will allow us in the future to obtain a good approximation to the full N³LO cross section.
- The inclusive gluon fusion cross section might get within reach in the next few years!

Conclusion

- We have developed a new technique to compute the threshold expansion of real emission diagrams.
- Application to Higgs@N3LO has allowed us to obtain the first two terms in the soft expansion of the triple real contribution.
 - ➔ Possible to obtain more terms in the expansion.
 - ➔ Most complicated contribution under control!
- Extension of this approach to mixed real-virtual integrals will allow us in the future to obtain a good approximation to the full N3LO cross section.
- The inclusive gluon fusion cross section might get within reach in the next few years!